Markov process:

A Markov process is a stochastic process that satisfies the Markov property[1] (sometimes characterized as "memorylessness"). In simpler terms, it is a process for which predictions can be made regarding future outcomes based solely on its present state and--most importantly--such predictions are just as good as the ones system, its future and past states are independent.

A Markov chain is a type of Markov process that has either a discrete state space or a discrete index set (often that could be made knowing the process's full history.[11] In other words, conditional on the present state of the representing time), but the precise definition of a Markov chain varies.[12] For example, it is common to define a Markov chain as a Markov process in either discrete or continuous time with a countable state space (thus regardless of the nature of time),[13][14][15][16] but it is also common to define a Markov chain as having discrete time in either countable or continuous state space (thus regardless of the state space).[12]

A Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event.[1][2][3] In continuous-time, it is known as a Markov process. It is named after the Russian mathematician Andrey Markov.

Markov chains have many applications as statistical models of real-world processes,[1][4][5][6] such as studying cruise control systems in motor vehicles, queues or lines of customers arriving at an airport, currency exchange rates and animal population dynamics.[7]

Markov processes are the basis for general stochastic simulation methods known as Markov chain Monte Carlo, which are used for simulating sampling from complex probability distributions, and have found application in Bayesian statistics and artificial intelligence.[7][8][9]

The adjective Markovian is used to describe something that is related to a Markov process.[1][10]

Types of Markov chains:

The system's state space and time parameter index need to be specified. The following table gives an overview of the different instances of Markov processes for different levels of state space generality and for discrete time v. continuous time:

	Countable state space	Continuous or general state space
Discrete-time	(discrete-time) Markov chain on a countable or finite state space	Markov chain on a measurable state space (for example, Harris chain)
Continuous-time	Continuous-time Markov process or Markov jump process	Any continuous stochastic process with the Markov property (for example, the Wiener process)

Note that there is no definitive agreement in the literature on the use of some of the terms that signify special cases of Markov processes. Usually the term "Markov chain" is reserved for a process with a discrete set of times, that is, a discrete-time Markov chain (DTMC),[1][17][17] but a few authors use the term "Markov process" to refer to a continuous-time Markov chain (CTMC) without explicit mention.[18][19][20] In addition, there are other extensions of Markov processes that are referred to as such but do not necessarily fall within any of these four categories (see Markov model). Moreover, the time index need not necessarily be real-valued; like with the state space, there are conceivable processes that move through index sets with other mathematical constructs. Notice that the general state space continuous-time Markov chain is general to such a degree that it has no designated term.

While the time parameter is usually discrete, the state space of a Markov chain does not have any generally agreed-on restrictions: the term may refer to a process on an arbitrary state space.[21] However, many applications of Markov chains employ finite or countably infinite state spaces, which have a more straightforward statistical analysis. Besides time-index and state-space parameters, there are many other variations, extensions and generalizations (see Variations). For simplicity, most of this article concentrates on the discrete-time, discrete state-space case, unless mentioned otherwise.

Transitions:

The changes of state of the system are called transitions.[1] The probabilities associated with various state changes are called transition probabilities. The process is characterized by a state space, a transition matrix describing the probabilities of particular transitions, and an initial state (or initial distribution) across the state space. By convention, we assume all possible states and transitions have been included in the definition of the process, so there is always a next state, and the process does not terminate.

A discrete-time random process involves a system which is in a certain state at each step, with the state changing randomly between steps.[1] The steps are often thought of as moments in time, but they can equally well refer to physical distance or any other discrete measurement.

Formally, the steps are the integers or natural numbers, and the random process is a mapping of these to states.[22] The Markov property states that the conditional probability distribution for the system at the next step (and in fact at all future steps) depends only on the current state of the system, and not additionally on the state of the system at previous steps.

Since the system changes randomly, it is generally impossible to predict with certainty the state of a Markov chain at a given point in the future.[22] However, the statistical properties of the system's future can be predicted.[22] In many applications, it is these statistical properties that are important.

Examples:

Random walks based on integers and the gambler's ruin problem are examples of Markov processes.[40][41] Some variations of these processes were studied hundreds of years earlier in the context of independent variables.[42][43][44] Two important examples of Markov processes are the Wiener process, also known as the Brownian motion process, and the Poisson process,[27] which are considered the most important and central stochastic processes in the theory of stochastic processes.[45][46][47] These two processes are Markov processes in continuous time, while random walks on the integers and the gambler's ruin problem are examples of Markov processes in discrete time.[40][41]

A famous Markov chain is the so-called "drunkard's walk", a random walk on the number line where, at each step, the position may change by +1 or −1 with equal probability. From any position there are two possible transitions, to the next or previous integer. The transition probabilities depend only on the current position, not on the manner in which the position was reached. For example, the transition probabilities from 5 to 4 and 5 to 6 are both 0.5, and all other transition probabilities from 5 are 0. These probabilities are independent of whether the system was previously in 4 or 6.

Another example is the dietary habits of a creature who eats only grapes, cheese, or lettuce, and whose dietary habits conform to the following rules:

- It eats exactly once a day.
- If it ate cheese today, tomorrow it will eat lettuce or grapes with equal probability.
- If it ate grapes today, tomorrow it will eat grapes with probability 1/10, cheese with probability 4/10, and lettuce with probability 5/10.
- If it ate lettuce today, tomorrow it will eat grapes with probability 4/10 or cheese with probability 6/10. It will not eat lettuce again tomorrow.

This creature's eating habits can be modeled with a Markov chain since its choice tomorrow depends solely on what it ate today, not what it ate yesterday or any other time in the past. One statistical property that could be calculated is the expected percentage, over a long period, of the days on which the creature will eat grapes.

A series of independent events (for example, a series of coin flips) satisfies the formal definition of a Markov chain. However, the theory is usually applied only when the probability distribution of the next step depends non-trivially on the current state.

4.2.2 Formal Definitions

Definition 4.1. A *Markov chain* is a sequence of random variables $X_1, X_2, X_3, \ldots$ with the *Markov property*, namely that the probability of any given state X_n only depends on its immediate previous state X_{n-1}. Formally:

$$P(X_n = x \mid X_{n-1} = x_{n-1}, \ldots, X_1 = x_1) = P(X_n = x \mid X_{n-1} = x_{n-1})$$

where $P(A \mid B)$ is the probability of A given B.

The possible values of X_i form a countable set S called the *state space* of the chain. If the state space is finite, and the Markov chain time-homogeneous (i.e. the transition probabilities are constant in time), the transition probability distribution can be represented by a matrix $\mathrm{P} = (p_{ij})_{i,j \in S}$, called the *transition matrix*, whose elements are defined as:

$$p_{ij} = P(X_n = j \mid X_{n-1} = i)$$

Let $\mathrm{x}^{(n)}$ be the *probability distribution* at time step n, i.e. a vector whose i-th component describe the probability of the system to be in state i at time state n:

$$\mathrm{x}_i^{(n)} = P(X_n = i)$$

Transition probabilities can be then computed as power of the transition matrix:

$$\begin{aligned}
\mathrm{x}^{(n+1)} &= \mathrm{P} \cdot \mathrm{x}^{(n)} \\
\mathrm{x}^{(n+2)} &= \mathrm{P} \cdot \mathrm{x}^{(n+1)} = \mathrm{P}^2 \cdot \mathrm{x}^{(n)} \\
\mathrm{x}^{(n)} &= \mathrm{P}^n \cdot \mathrm{x}^{(0)}
\end{aligned}$$